

Passive Stability of a Spinning Skylab

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An analytical investigation is made of the rotational dynamics of a torque-free rigid body which must spin about a principal axis of intermediate moment of inertia. Massless flexible booms with discrete tip masses are attached to the body, and the physical characteristics of these appendages are determined so that the rotational dynamics of the composite body will be stable. The parameter plane stability technique is applied to linearized equations of motion representing the model with the physical characteristics of a spinning Skylab. The transient response to external disturbances is predicted in terms of appendage characteristics and vehicle spin rate and then calculated explicitly in a design example.

Nomenclature

a_p	= coefficients of wobble characteristic equation (p ranges from 0-4)
$d_i = 2m\zeta_i\omega_{ni}$	= structural damping coefficient in i direction
E	= unit matrix of same dimensions as F matrix
F	= matrix associated with vector matrix state equation $\dot{x}' = Fx$
I_i^*	= principal moment of inertia of rigid-core body about i body-fixed coordinate
I_1, I_2, I_3	= principal moment of inertia of body about i coordinate $I_1^* + 2m\Gamma^2$, I_2^* , $I_3^* + 2m\Gamma^2$, respectively
K_1, K_2	= ratios of inertia $(I_2 - I_3)/I_1$ and $(I_3 - I_1)/I_2$, respectively
k_i	= stiffness coefficient characterizing non-rotating boom stiffness
\mathcal{L}^{-1}	= inverse Laplace transform
m	= tip mass of boom
T_i	= applied torque about i coordinate
t	= time
$u_i = u_i^1 - u_i^2$	= skew symmetric mode of elastic deformation
u_i^m	= displacement of m tip mass from spinning steady state in i direction ($m = 1, 2$)
w_i	= perturbation (about spinning steady state) velocity about i coordinate
X_p, Y_p	= real and imaginary parts of λ^p , respectively
$x(\tau) = [w_1, w_2, \mu_3, \mu_3']^T$	= wobble-motion state vector
α	= a positive constant such that $k_2 = \alpha k_1$
Γ	= steady-state boom dimension in 2-axis direction from center of mass to tip mass
$\gamma_i = 2m\Gamma^2/I_i$	= dimensionless inertia ratio
$\Delta_i = d_i/m\Omega$	= dimensionless damping ratio
$\xi_i = \eta/(\nu^2 + \eta^2)^{1/2}$	= damping ratio of boom in i direction
$\lambda = s/\Omega = \eta + i\nu$	= normalized Laplace operator
$\mu_i = u_i/2\Gamma$	= generalized skew symmetric coordinate
$\nu_n = (\nu^2 + \eta^2)^{1/2}$	= nondimensional natural frequency
$\sigma_i^2 = k_i/m\Omega^2$	= dimensionless natural frequency coefficient of boom
$\tau = \Omega t$	= dimensionless time
$\Phi(\tau)$	= state transition matrix
Φ_{gr}	= elements of state transition matrix

Ω	= steady-state spin rate about 3 axis
$\omega = [w_1, w_2, \Omega + w_3]^T$	= angular velocity vector
ω_{ni}	= natural frequency of boom in i direction
$(\cdot)'$	= differentiation with respect to τ
Subscript i	= index referring to three body-fixed coordinates ($i = 1, 2, 3$)

Introduction

If it should be desired to provide artificial gravity by spinning the Skylab, the solar panels still must point toward the sun. The vehicle would be required to spin about a principal axis of intermediate moment of inertia, which cannot be done stably. That axis can be made the axis of maximum moment of inertia by deploying masses on the tips of extendable booms in a direction perpendicular to the spin axis. A simplified model of the modified spinning Skylab is described, and analyses of the stability of motion and the rotational dynamics are presented.

Model Definition and Equations of Motion

The geometrically complex Skylab¹ (Fig. 1) is simplified to make it analytically tractable. The vehicle is modeled (Fig. 2) as a single rigid-core body with principal moments of inertia I_i^* with $I_1^* < I_3^* < I_2^*$. Attached to the core body are two flexible massless booms, each of length Γ and each having a tip mass m . In the steady state, the principal moments of inertia of the entire vehicle coincide with the body-fixed axes and are designated I_i with $I_1 < I_2 < I_3$. The vehicle is assumed to have a steady-state spin velocity Ω about the 3 axis, and the angular velocity vector is written in body-fixed coordinates as $\omega = [w_1, w_2, \Omega + w_3]^T$ where w_i represents small perturbations about the steady state—i.e.,

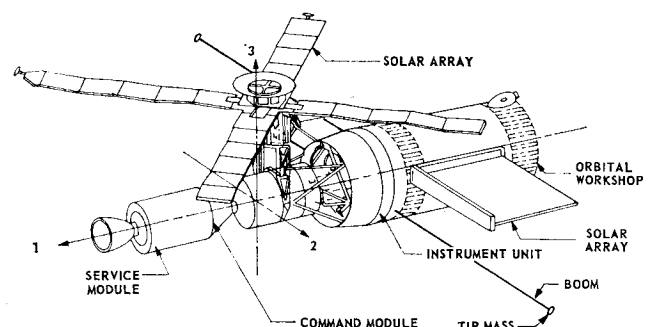


Fig. 1 Skylab.

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by the shaded region of Fig. 3. Also, the effect of boom length and tip mass may be seen by considering their effect on K_1 .

Similarly, stability of the in-plane motion may be determined by examining the characteristic equation obtained from Eqs. (2)

$$\begin{aligned} & \lambda \{ (1 - \gamma_3) \lambda^4 + [\Delta_1 + \Delta_2(1 - \gamma_3)] \lambda^3 \\ & + [\sigma_1^2 + \sigma_2^2(1 - \gamma_3) + 3(1 - \gamma_3) + \Delta_1 \Delta_2] \lambda^2 \\ & + [\Delta_1(\sigma_2^2 - 1 + 4\gamma_3) + \Delta_2 \sigma_1^2] \lambda + \sigma_1^2(\sigma_2^2 - 1 + 4\gamma_3) \} \\ & = 0 \end{aligned} \quad (10)$$

If it is assumed that the stiffness coefficient k_2 is α times as great as k_1 and k_3 , where α is a positive constant, $1/\alpha \ll 1$, and that $\zeta_1 = \zeta_3$, the application of D -composition to Eq. (10) yields the boundary associated with the origin of the λ -plane

$$\sigma_2^2 = 0 \text{ and } \sigma_2^2 = 1 - 4\gamma_3 \quad (11)$$

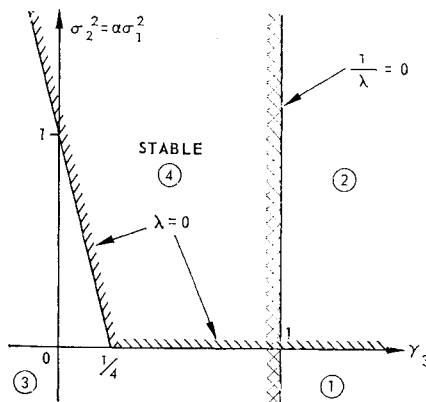
The boundary associated with infinity on the λ -plane is $\gamma_3 = 1$. The boundary associated with the imaginary axis of the λ -plane yields the same two boundaries plus an additional constraint, $\Delta_2 = 0$. No additional boundaries arise from the $J = 0$ condition. The resulting region of stability is shown on a γ_3, σ_2^2 parameter plane (Fig. 4), which shows the effects of the boom characteristics on the stability of the in-plane motion.

Wobble-Motion Dynamics

Once stability has been assured, the transient dynamics of the wobble motion are of interest. The character of this motion may be investigated by using the parameter plane technique.² From the parameters in characteristic equation (3), inertia ratios $K_1 K_2$ and K_2 are selected as two adjustable parameters of interest to the designer—i.e., the effect of various values of $K_1 K_2$ and K_2 on the wobble-motion dynamics will be determined. This is done by finding the relationship between these two parameters and the roots of Eq. (3). The relationship may be shown by mapping a convenient contour from the λ -plane onto the $K_1 K_2, K_2$ parameter plane. The contour selected is the line

$$\lambda = -\zeta \nu_n + i \nu_n (1 - \zeta^2)^{1/2} \quad (12)$$

where $\nu_n = \nu(1 - \zeta^2)^{-1/2}$. Any root of Eq. (3) lying on this contour will have a specified damping ratio ζ . This line is mapped onto the parameter plane by substituting Eq. (12) into Eq. (3), which may be written in the form



NOTES:

1. SEE NCTE 2, FIG. 3

2. ADDITIONAL CONSTRAINT: $\Delta_2 > 0$

Fig. 4 Stability region for in-plane motion.

$$\sum_{p=0}^4 (a_p K_1 K_2 + b_p K_2 + c_p) \lambda^p = 0 \quad (13)$$

where values for coefficients a_p, b_p, c_p are found in Table 1. If λ^p is written as the sum of its real and imaginary parts,

$$\lambda^p = X_p + i Y_p \quad (14)$$

values of X_p and Y_p may be found from the recurrence formulas,

$$\begin{aligned} X_{p+1} + 2\zeta \nu_n X_p + \nu_n^2 X_{p-1} &= 0 \\ Y_{p+1} + 2\zeta \nu_n Y_p + \nu_n^2 Y_{p-1} &= 0 \end{aligned} \quad (15)$$

where it is seen from Eqs. (12) and (14) that $X_0 = 1, Y_0 = 0, X_1 = -\zeta \nu_n$, and $Y_1 = \nu_n(1 - \zeta^2)^{1/2}$. Substituting Eq. (14) into Eq. (13) and separating the resulting real and imaginary parts, one obtains two simultaneous algebraic equations that may be solved for parameters $K_1 K_2$ and K_2

$$\begin{aligned} K_1 K_2 &= (B_1 C_2 - B_2 C_1)/J, \quad K_2 = (A_2 C_1 - A_1 C_2)/J \\ J &= A_1 B_2 - A_2 B_1 \end{aligned} \quad (16)$$

where

$$A_1 = \sum_{p=0}^4 a_p X_p, \quad B_1 = \sum_{p=0}^4 b_p X_p, \quad C_1 = \sum_{p=0}^4 c_p X_p$$

and

$$A_2 = \sum_{p=0}^4 a_p Y_p, \quad B_2 = \sum_{p=0}^4 b_p Y_p, \quad C_2 = \sum_{p=0}^4 c_p Y_p \quad (17)$$

Equations (16) define the parameter plane contour corresponding to Eq. (12) in the λ -plane for a chosen value of ζ and coefficients a_p, b_p, c_p . Each contour is plotted as a function of the nondimensional natural frequency ν_n .

The real roots of the system may also be mapped from the λ -plane onto the parameter plane. Each selected value η of a real root is substituted for λ in characteristic Eq. (13). The resulting equation will define a contour on the parameter plane representing the selected location η of the real root.

The response of the system in terms of variables w_1, w_2, μ_3 may be found by casting Eqs. (1) in the form

$$\dot{\mathbf{x}} = \mathbf{F} \mathbf{x} \quad (18)$$

where the vector \mathbf{x} is defined as

$$\mathbf{x} = [w_1, w_2, \mu_3, \mu_3']^T \quad (19)$$

and the matrix \mathbf{F} is

$$\mathbf{F} = \frac{1}{(1 - \gamma_1)} \begin{bmatrix} 0 & (\gamma_1 + K_1) & \gamma_1 \sigma_3^2 & \gamma_1 \Delta_3 \\ K_2(1 - \gamma_1) & 0 & 0 & 0 \\ 0 & 0 & 0 & (1 - \gamma_1) \\ 0 & -(K_1 + 1) & (\gamma_1 - \sigma_3^2 - 1) & -\Delta_3 \end{bmatrix} \quad (20)$$

The response $\mathbf{x}(\tau)$ may then be found from

$$\mathbf{x}(\tau) = \Phi(\tau) \mathbf{x}(0) \quad (21)$$

where $\mathbf{x}(0)$ is the initial condition vector and $\Phi(\tau)$ is the state transition matrix

Table 1 Coefficients of wobble; characteristic equation

p	a_p	b_p	c_p
0	$-(\sigma_3^2 + 1)$	$-\gamma_1$	0
1	$-\Delta_3$	0	0
2	-1	$-\gamma_1$	$1 + \sigma_3^2 - \gamma_1$
3	0	0	Δ_3
4	0	0	$1 - \gamma_1$

$$\Phi(\tau) = e^{F\tau} = \mathcal{L}^{-1}\{[\lambda E - F]^{-1}\} \quad (22)$$

In evaluating $\Phi(\tau)$, the roots of the characteristic equation (3) are the roots of the determinant $|\lambda E - F|$ and may be taken directly from the parameter plane (for example, if initial conditions exist only on w_1 and w_2 —i.e.

$$\mathbf{x}(0) = [x_{10}, x_{20}, 0, 0]^T \quad (23)$$

the $w_1(\tau)$ response is found from

$$w_1(\tau) = \mathcal{L}^{-1}[\Phi_{11}(\lambda)x_{10} + \Phi_{21}(\lambda)x_{20}] \quad (24)$$

where

$$\begin{aligned} \Phi_{11}(\lambda) &= \lambda[\lambda^2 + \Delta_3\lambda/(1-\gamma_1) \\ &\quad + (1 + \sigma_3^2 - \gamma_1)/(1-\gamma_1)]/|\lambda E - F| \\ \Phi_{21}(\lambda) &= K_2\Phi_{11}(\lambda)/\lambda \end{aligned} \quad (25)$$

Example

The model described by Eqs. (1) and (2) is used to represent a spinning Skylab. Representative values for the coefficients in Eq. (3) are given in Table 2. If the values are substituted

Table 2 Physical characteristics of Skylab

$I_1^* = 1.01 \times 10^6 \text{ kg m}^2$	$\Gamma = 23.3 \text{ m}$	$k_2 = 7.4 \times 10^4 \text{ N/m}$
$I_2^* = 6.90 \times 10^6 \text{ kg m}^2$	$m = 227 \text{ kg}$	$d_1 = 0.04(k_1 m)^{1/2}$
$I_3^* = 6.85 \times 10^6 \text{ kg m}^2$	$k_1 = k_3 = 146 \text{ N/m}$	$\Omega = 0.6 \text{ s}^{-1}$

into Eqs. (16), ζ -contours may be plotted on the K_1K_2 , K_2 parameter plane as functions of ν_n (Fig. 5). Stability boundaries defined by Eqs. (4, 7, and 9) are also plotted in Fig. 5. Real root boundaries are found to be outside the stable region and hence are of no interest in the Skylab problem.

A typical design point for Skylab is shown on the parameter plane. It corresponds to the physical characteristics shown in Table 2. The corresponding roots of the characteristic equation are seen to have damping ratios ζ and nondimensional natural frequencies ν_n of

$$\begin{aligned} \zeta &= 0.002, & \nu_n &= 0.2854 \\ \text{and} & & & \\ \zeta &= 0.01863, & \nu_n &= 1.7612 \end{aligned} \quad (26)$$

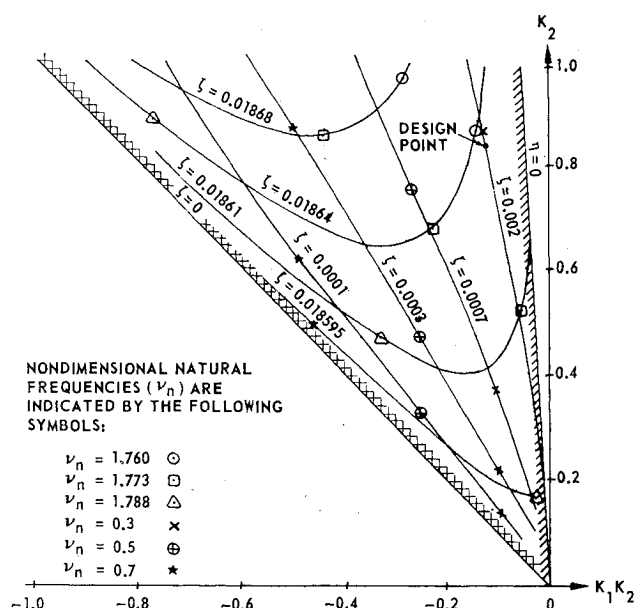


Fig. 5 K_2 vs K_1K_2 parameter plane plot.

If the values of Table 2 and Eqs. (26) are substituted into Eq. (21), the response for the Skylab model is found. For example, from Eq. (24)

$$\begin{aligned} w_1(\tau) &= 1.0978e^{-0.0005672\tau} \cos(0.2854\tau + 0.3274)w_1(0) \\ &\quad + 0.09359e^{-0.03281\tau} \cos(1.761\tau + 4.1270)w_2(0) \end{aligned} \quad (27)$$

For $w_1(0) = w_2(0) = 0.001$, the response $w_1(\tau)$ is plotted in Fig. 6. Similarly, $w_2(\tau)$ and $\mu_3(\tau)$ are also determined and shown in Figs. 7 and 8.

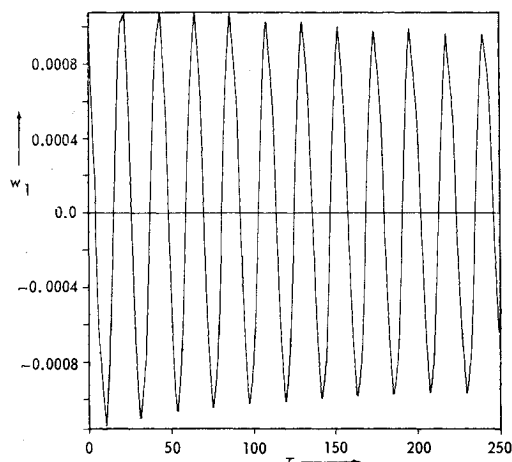


Fig. 6 w_1 vs τ .

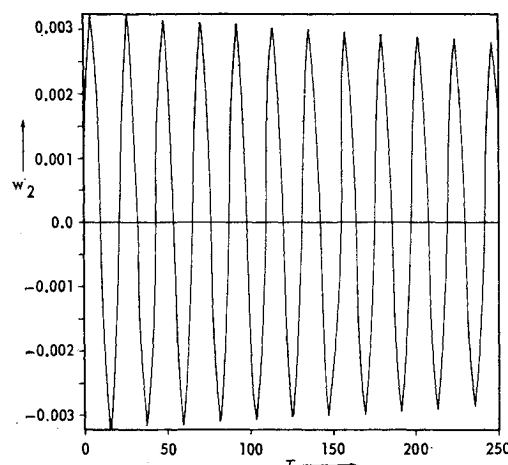


Fig. 7 w_2 vs τ .

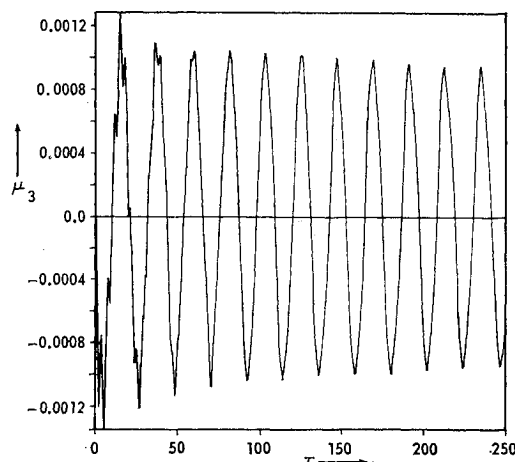


Fig. 8 μ_3 vs τ .

Conclusion

It has been shown that it is possible to stabilize passively the motion of a simplified model of a spinning Skylab by deploying flexible booms, thus altering the moments of inertia. Analytical results indicate the required boom-stiffness properties for given vehicle mass properties and spin rates to achieve passive stability. Furthermore, the use of the simplified model leads to results amenable to physical interpretation.

To gain confidence that these results will apply to the actual Skylab, an additional step is being implemented. A detailed digital simulation model of the spinning Skylab vehicle has

been developed at the Marshall Space Flight Center, and results compare favorably with those of the simplified model.

References

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Supersonic Dynamic Stability Experiments on the Space Shuttle

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A wind-tunnel study was performed of the longitudinal dynamic stability of the shuttle spacecraft at supersonic speeds. In particular, the study included the determination of the damping-in-pitch characteristics of the orbiter and of the booster, both separately and in a mated launch configuration, and the investigation of the effect of a simulated rocket exhaust plume and of the static and dynamic interference during abort separation. Static interference is defined here as caused by the proximity of the second vehicle when that vehicle is stationary, whereas dynamic interference denotes the same effect, but with the other vehicle performing an oscillatory motion. It was found that, in general, the static interference effects were small, whereas, for the case of the orbiter and the booster performing synchronous oscillations, the dynamic interference effect was very large and could lead to a strongly negative damping of the orbiter.

Introduction

THE unconventional configurations employed for the space shuttle and its often rather unusual flight conditions render the analytical prediction of its aerodynamic characteristics extremely difficult. Thus in many cases the experimental data constitute the sole source of information. One such area where a heavy dependence on experiments is necessary involves the prediction of the dynamic stability characteristics of the shuttle. Although often not of critical importance for the preliminary design, this information usually is required for the complete performance analysis. In addition, there are several flight conditions, such as the ascent of the launch configuration, some phases of the abort separation, and the transitional flight of the orbiter, where the dynamic stability characteristics of the shuttle may very well prove to be significant in the final design and formulation of operational procedures.

This paper describes the dynamic stability experiments on the shuttle, which were performed as part of the internal research program of the National Aeronautical Establishment

(NAE). In addition to discussing certain new concepts and new techniques which were specially developed for these experiments, the paper contains a representative selection of dynamic stability results for the shuttle, including certain unusual interference effects such as those occurring during an abort separation and in the presence of an exhaust plume. Oil-flow visualization pictures and high-speed shadowgraph movies are used to illustrate some of the complex flowfields involved in these cases.

Experimental Arrangement

The experiments were performed on 1/360 scale models of the fully reusable shuttle spacecraft, which were manufactured and supplied by NASA Langley Research Center. Models typical of both the low cross-range and the high cross-range configurations were investigated. The former were represented by a straight-wing orbiter and a straight-wing booster, and the latter by a delta-wing orbiter and swept-wing canard booster. Details of the two sets of models are given in Refs. 1-3. The pitching moment derivatives of both orbiters were referred to the total wing area and the wing mean aerodynamic chord, while a scaled-down standard reference area (10,000 ft²) and reference length (200 ft) were used for the two boosters. The distance of the axis of oscillation (and the moment reference point) from the model nose, referred to the model length, was 0.612 and 0.610 for the straight-wing orbiter and booster, respectively, and 0.456 and 0.465 for the delta-wing orbiter and booster.

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